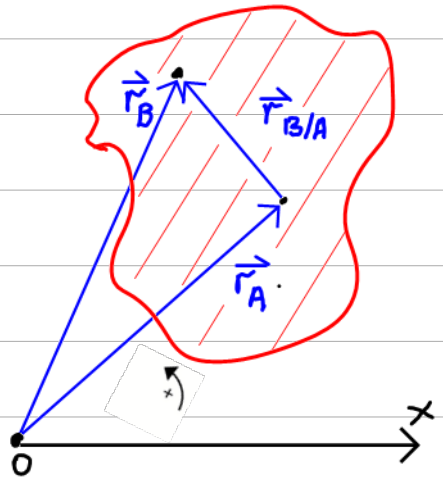
Meriam 5/3 – 5/5 : General Plane Motion – Velocity

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$

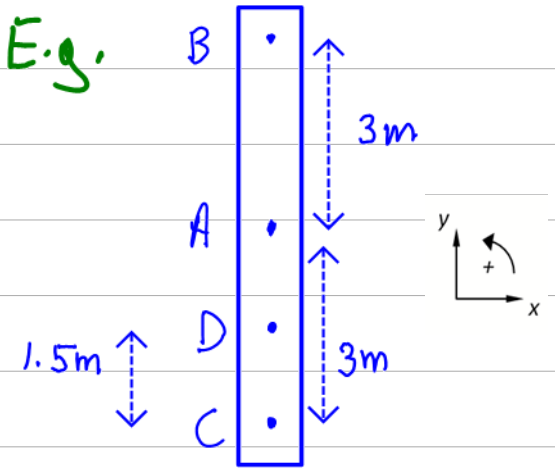
magnitude ωr_{AB} direction $\perp r_{AB}$



where $|\omega| = \dfrac{|\vec{V}_P - \vec{V}_Q|}{r_{PQ}}$

where P & Q are any 2 points on the body

E.g.



Given: $\vec{V}_B = 4\hat{i} + 3\hat{j}$ m/s

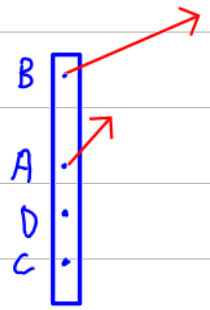
$\vec{V}_A = 2\hat{i} + 3\hat{j}$ m/s.

Determine: ω, $\vec{V}_D$, $\vec{V}_C$

Answer:

$$|\omega| = \frac{|\vec{V}_A - \vec{V}_B|}{|r_{AB}|} = \frac{|(2\hat{i}+3\hat{j}) - (4\hat{i}+3\hat{j})|}{|3 \text{ meters}|}$$

$$\omega = \frac{|-2\hat{i}|}{3} = 0.667\ s^{-1} \circlearrowright = -0.667\ s^{-1}$$



$$\vec{V}_C = \vec{V}_B + \vec{V}_{C/B} = (4\hat{i} + 3\hat{j}) + |\omega|\,|r_{BC}|\,(-\hat{i}) = 3\hat{j} \text{ m/s}$$

(0.667, 6)

$$\vec{V}_D = \vec{V}_C + \vec{V}_{D/C} = 3\hat{j} + |\omega|\,|r_{DC}|\,(\hat{i}) = 1\hat{i} + 3\hat{j} \text{ m/s}$$

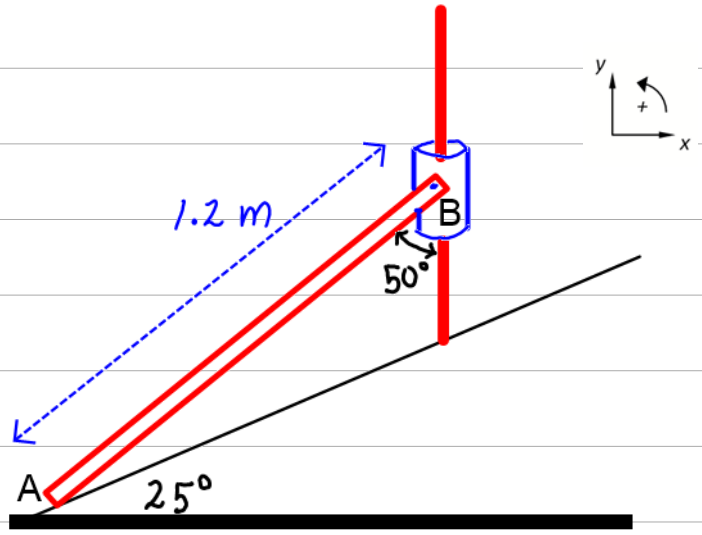
(0.667, 1.5)

(or $\vec{V}_D$ is average of $\vec{V}_A$ & $\vec{V}_C$)

Eg.

Given:

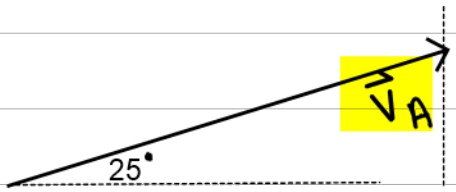
$\vec{V}_B = 1.5\,\hat{j}$ m/s.

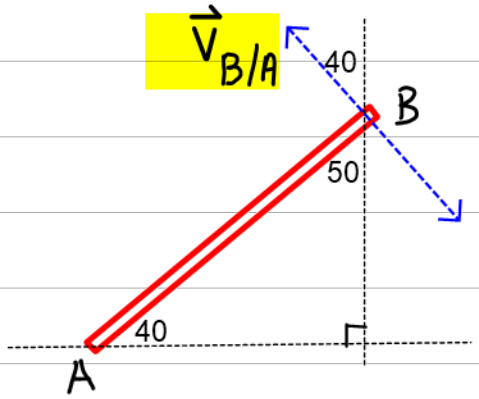


Find: ω_{AB}

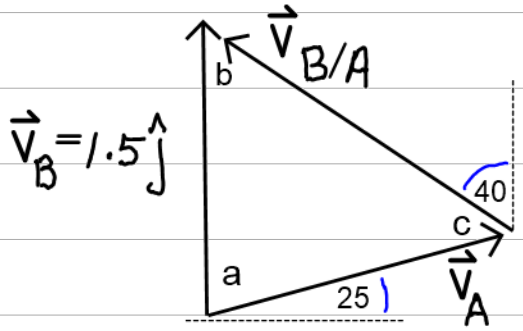
$|\vec{V}_A|$

$\vec{V}_B = 1.5\uparrow$





$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$



$\angle a = 90 - 25 = 65°$

$\angle b = 40°$

$\angle c = 180 - a - b = 75°$

sine law:

$$\frac{|\vec{V}_B|}{\sin(c)} = \frac{|\vec{V}_A|}{\sin(b)} \qquad (|\vec{V}_B| = 1.5,\ c = 75,\ b = 40)$$

$$\rightarrow \boxed{|\vec{V}_A| = 0.998 \text{ m/s}}$$

sine law again:

$$\frac{|\vec{V}_B|}{\sin(c)} = \frac{|\vec{V}_{B/A}|}{\sin(a)} \qquad (|\vec{V}_B| = 1.5,\ c = 75,\ a = 65)$$

$$\rightarrow \boxed{|\vec{V}_{B/A}| = 1.41 \text{ m/s}}$$

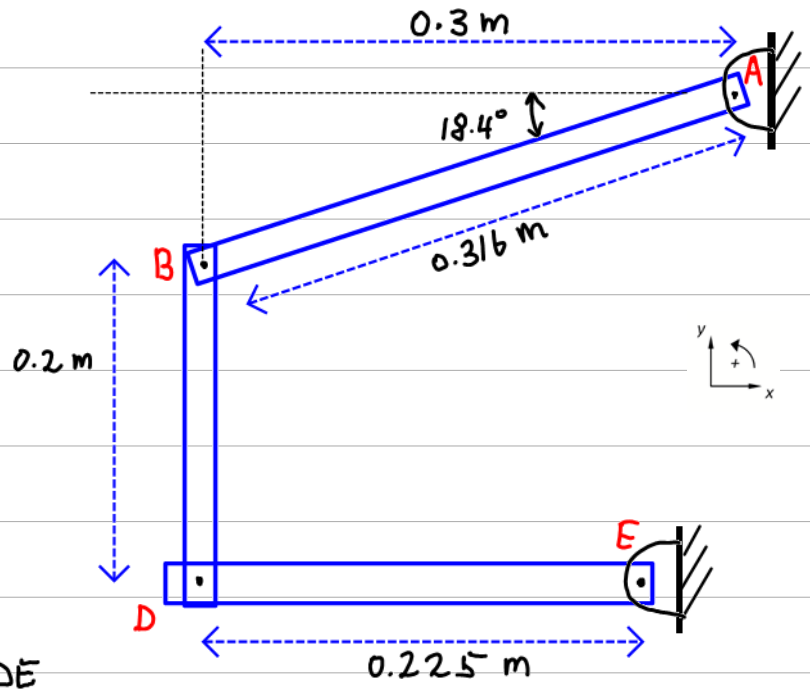
but $|\vec{V}_{B/A}| = |\omega\, r_{AB}|$ $\qquad (|\vec{V}_{B/A}| = 1.41,\ r_{AB} = 1.2)$

$$\boxed{\omega = +1.173\ s^{-1}}$$

Eg.

Given: $\omega_{AB} = 3\ s^{-1}$

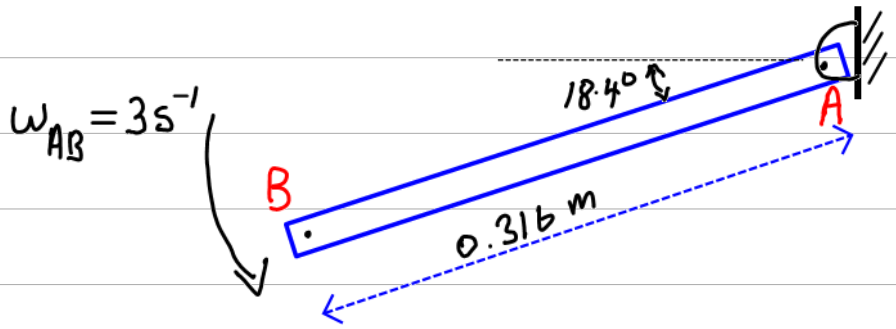
Find: ω_{BD} , ω_{DE}



→ Take a sequenced approach

ω_{AB} ① → $\vec{v}_B$ ② → ω_{BD} & $\vec{v}_D$ ③ → ω_{DE}

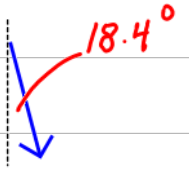
①



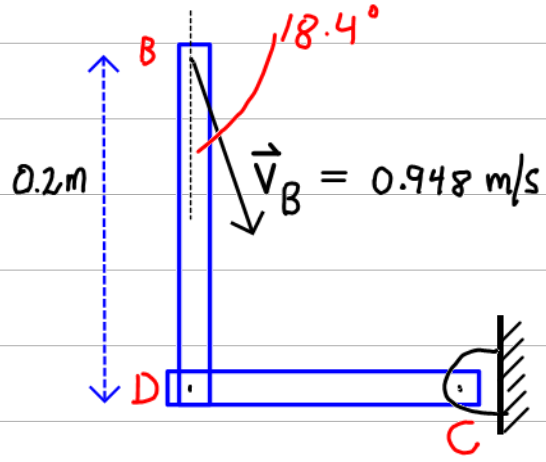
Fixed-axis rotation:

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_B = |\omega_{AB}\, r_{AB}| \downarrow = (3)(0.316) = 0.948 \text{ m/s}$$



step ②

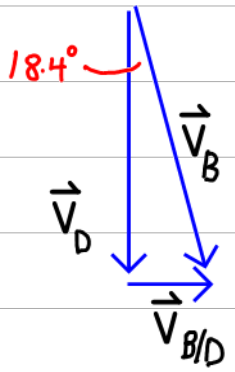


Note :

(i) D can only move vertically

(ii) $\vec{V}_{B/D}$ is horizontal

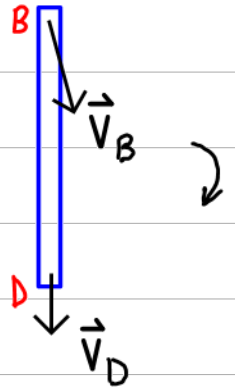
$$\vec{V}_B = \vec{V}_D + \vec{V}_{B/D}$$



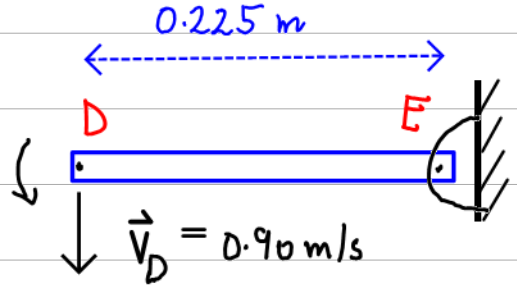
$$\rightarrow \quad \vec{V}_D = 0.948 \cos 18.4^\circ = \boxed{0.90 \text{ m/s} \downarrow}$$

$$\& \quad \vec{V}_{B/D} = 0.948 \sin 18.4^\circ = 0.30 \text{ m/s} \rightarrow$$

$$\& \quad \omega_{BD} = |\vec{V}_{B/D} / r_{BD}| = \frac{0.3}{0.2} = \boxed{-1.5 \text{ s}^{-1}}$$ (direction from diagram)



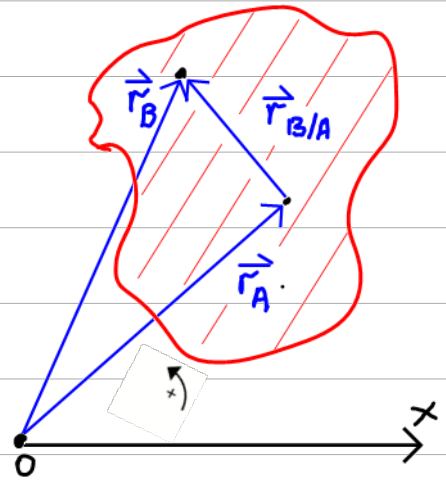
step ③



$$|\vec{V}_D| = |\vec{V}_E^{\,0} + \vec{V}_{D/E}| \quad \rightarrow \quad 0.90 = |(\omega_{DE})(0.225)|$$

$$\rightarrow \quad \boxed{\omega_{DE} = +4 \text{ s}^{-1}}$$ (direction from diagram)

Summary



- $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

magnitude ωr_{AB} direction $\perp r_{AB}$

where $|\omega| = \dfrac{|\vec{v}_P - \vec{v}_Q|}{r_{PQ}}$

where P & Q are any 2 points on the body

- Solve as a vector triangle

Next Lecture

- Read Meriam 5/3 – 5/6